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Fourth Semester B.E. Degree Examination, Dec.2016/Jan.2017
Engineering Mathematics - IV

Time: 3 hrs.   Max. Marks:100

Note: *Answer any FIVE full questions, selecting atleast TWO questions from each part.*

PART – A

1   a. Using Taylor series method, solve $\dfrac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ at $x = 0.2$.   **(06 Marks)**

   b. Using Runge – Kutta method of fourth order solve for $y(0.1)$, $y(0.2)$ given that
   $\dfrac{dy}{dx} = y(x + y)$, $y(0) = 1$.   **(07 Marks)**

   c. Given $\dfrac{dy}{dx} = x^2 (1 + y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$,
   evaluate $y(1.4)$ by Milne's Predictor – Corrector method.   **(07 Marks)**

2   a. Approximate y and z at $x = 0.1$, using Picard's method for the solution of the equations
   $\dfrac{dy}{dx} = z$, $\dfrac{dz}{dx} = x^3 (y + z)$, given that $y(0) = 1$ and $z(0) = \frac{1}{2}$.   **(06 Marks)**

   b. Using Runge – Kutta fourth order method to solve $y'' = xy' - y$, $y(0) = 3$, $y'(0) = 0$, find y
   and z at $x = 0.1$.   **(07 Marks)**

   c. Apply Milne's method to compute $y(0.4)$ given that $y'' + xy' + y = 0$ and the values
   $y(0) = 1$, $y(0.1) = 0.995$, $y(0.2) = 0.9801$, $y(0.3) = 0.956$, $y'(0) = 0$,
   $y'(0.1) = -0.0995$, $y'(0.2) = -0.196$, $y'(0.3) = -0.2867$.   **(07 Marks)**

3   a. Prove that the C – R equations in polar form.   **(06 Marks)**

   b. Show that $f(z) = z^n$, where n is a positive integer is analytic and hence find its derivative.   **(07 Marks)**

   c. If $\phi + i\Psi$ represents the complex potential of an electrostatic field where
   $\Psi = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, find ϕ.   **(07 Marks)**

4   a. Find the Bilinear transformation which maps the points $1, i - 1$ into $0, 1, \infty$.   **(06 Marks)**

   b. State and prove the Cauchy's integral formula.   **(07 Marks)**

   c. Evaluate $\displaystyle\int_c \dfrac{e^{2z}}{(z+1)(z-2)} dz$, where $c : |z| = 3$.   **(07 Marks)**

PART – B

5   a. Find the solution of the Laplace's equation in cylindrical system leading to Bessel's
   differential equation.   **(06 Marks)**

   b. Derive Rodrigue's formula
   $P_n(x) = \dfrac{1}{2^n n!} \dfrac{d^n}{dx^n} (x^2 - 1)^n$.   **(07 Marks)**

   c. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms Legendre polynomials.   **(07 Marks)**

6 a. Define the Empherical and Axiomatic definition of probability and give an example of each.
(06 Marks)

b. Of the cigarette smoking population 70% are men and 30% are women, 10% of these men and 20% of these women smoke wills. What is the probability that person seen smoking a wills will be a man?
(07 Marks)

c. The chance that a doctor will diagnose a disease correctly is 60%. The chance that a patient will die after correct diagnose is 40% and the chance of death by wrong diagnosis is 70%. If a patient dies, what is the chance that his disease was correctly diagnosed?
(07 Marks)

7 a. Derive the mean and variance of Binomial distribution.
(06 Marks)

b. If x is an exponential distribution with mean 4, evaluate i) $P(0 < x < 1)$ ii) $P(x > 2)$ and iii) $P(-\infty < x < 10)$.
(07 Marks)

c. The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) less than 65 ii) More than 75 and   iii) between 65 and 75.
(07 Marks)

8 a. Define the following terms :
i) Type I – error and Type II – error   ii) Level of significance.
(06 Marks)

b. A certain stimulus administered to each of the 12 patients resulted in the following :
Change in blood pressure 5, 2, 8, -1, 3, 0, 6, -2, 1, 5, 0, 4, can it be concluded that the stimulus will increase the blood pressure? (t. 05 for 11 d.f = 2.201).
(07 Marks)

c. The theory predicts the proportion of beans in the four groups G_1, G_2, G_3, G_4 should be in the ratio 9:3:3:1. In an experiment with 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (at 5% LOS for 3 d.f = 7.815).
(07 Marks)

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